ON SMALL PERTURBATIONS OF CONVECTIVE MOTION BETWEEN VERTICAL PARALLEL PLANES

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We consider a plane vertical layer of viscous incompressible liquid, enclosed between two infinite uniformly heated planes. It is well known that for arbitrarily small temperature difference between the planes convective motion of the liquid commences. The stability of the given motion was studied in [1 and 2], where the behavior of neutral perturbations was considered and it was shown that the critical point of steady motion is caused by monotonous and oscillatory perturbations at comparatively small values of the Grasshof number. Galerkin's method was applied to the study of the stability. The solutions consisited of linear combinations of basic functions which were polynomials satisfying the boundary conditions. A better result was obtained in [2], where four basic functions were chosen for approximation of the perturbations: two for the stream function and two for the temperature.

In the present paper we study the behavior of the normal perturbations in the liquid at rest and in the convective flow for a small temperature difference between the planes. The perturbations and their decrements are expanded in series with respect to a small parameter, the Grasshof number, and the usual method of the theory of perturbations is used to determine the eigenfunctions and the spectrum of the decrements taking into account second order corrections. We study the dependence of the spectrum of the Prandtl number P and it turns out that when $P \ll 1$ it passes over into the spectrum of the isothermal problem [3]. To establish the character of the intersection of the levels we use Galerkin's method in the approximation of four basic functions. The spectra of the decrements obtained by this method make it possible also to trace the onset of monotonous instability in a rather wide interval of the Prandtl number.

1. In an infinite vertical channel of width $2L$ with a constant temperature difference 2θ between the walls, a steady motion is set with profiles of dimensionless velocity $v_0 = {}^1/_6 (x^3 - x)$ and dimensionless temperature $T_0 = -x$. The direction of the axes is shown in Fig.1.

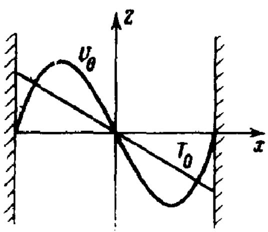

Fig. 1

Let us consider small plane perturbations of the velocity $\mathbf{u}(x, z, t)$ and of the temperature $\vartheta(x, z, t)$. The perturbation stream function $\Psi(x, z, t)$ is introduced by the relations $u_x = -d\Psi / dz$, $u_z = d\Psi / dx$. From the usual equations of free convection [4] we obtain for small plane perturbations the boundary problem

$$\Delta^2\Psi - \frac{\partial}{\partial t}\Delta\Psi + \frac{\partial\vartheta}{\partial x} = G\left(v_0\frac{\partial}{\partial z}\Delta\Psi - v_0''\frac{\partial\Psi}{\partial z}\right) \tag{1.1}$$

$$\left(\Delta = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial z^2}\right)$$

$$\frac{1}{P}\Delta\vartheta - \frac{\partial\vartheta}{\partial t} = G\left(v_0\frac{\partial\vartheta}{\partial z} - T_0'\frac{\partial\Psi}{\partial z}\right)$$

$$\left(G = \frac{g\beta\theta L^3}{\nu^2}, \quad P = \frac{\nu}{\varkappa}\right)$$

$$\frac{\partial\Psi}{\partial z} = \frac{\partial\Psi}{\partial x} = \vartheta = 0 \quad \text{for } x = \pm 1)$$

Here G is the Grasshof number; P is the Prandtl number; ν is the kinematic viscosity; g is the acceleration of free fall; β is the coefficient of thermal expansion. The equations are written in dimensionless variables. For units of distance, time, temperature and velocity we have taken the respective quantities L, L^2/ν, θ and $g\beta\theta L^2/\nu$.

We shall consider normal perturbations of the form

$$\Psi(x, z, t) = \Phi(x)\, e^{-(\lambda t + ikz)}, \qquad \vartheta(x, z, t) = T(x)\, e^{-(\lambda t + ikz)} \tag{1.2}$$

Here k is the real positive wave number, λ is the complex decrement, the real part of which characterizes the rate of growth (decay) of the perturbations, and the imaginary part their phase velocity. After substitution of (1.2) in (1.1) we obtain the equations for the amplitude of the perturbations

$$\Delta^2\Phi + \lambda\Delta\Phi + T' = aH\Phi, \frac{1}{P}\Delta T + \lambda T = -a(v_0 T + \Phi)$$

$$\Phi(\pm 1) = \Phi'(\pm 1) = T(\pm 1) = 0$$

$$\Delta = \frac{\partial}{\partial x^2} - k^2, \quad H\Phi = v_0''\Phi - v_0\Delta\Phi, \quad a = ikG \tag{1.3}$$

Here primes denote differentiation with respect to x.

2. For a small temperature difference between the walls, i.e. for small values of the parameter a, the method of perturbation theory can be used to determine the spectrum of the eigenvalues λ and the eigenfunctions of the boundary problem (1.3). We expand the amplitudes of the perturbation and the decrements λ in powers of the small parameter a

$$\Phi = \Phi^{(0)} + a\Phi^{(1)} + a^2\Phi^{(2)} + \ldots, \qquad T = T^{(0)} + aT^{(1)} + a^2T^{(2)} + \ldots$$

$$\lambda = \lambda^{(0)} + a^2\lambda^{(2)} + \ldots \tag{2.1}$$

In the expansion of the decrement λ only the even powers of a are retained, since λ does not change when a is replaced by $-a$ for odd unperturbed profiles of velocity v_0 and temperature T_0 (a similar situation exists also in the case of perturbations of isothermal flows with odd profiles [3]). Substituting the expansions (2.1) in the amplitude equations (1.3) and separating the terms with the parameter a raised to the zeroth power, first power, second power and so on, we obtain the equations of successive approximations

$$\Delta^2\Phi^{(0)} + \lambda^{(0)}\Delta\Phi^{(0)} + T^{(0)\prime} = 0, \qquad P^{-1}\Delta T^{(0)} + \lambda^{(0)}T^{(0)} = 0 \tag{2.2}$$

$$\Delta^2\Phi^{(1)} + \lambda^{(0)}\Delta\Phi^{(1)} + T^{(1)\prime} = H\Phi^{(0)}, \qquad P^{-1}\Delta T^{(1)} + \lambda^{(0)}T^{(1)} = -v_0T^{(0)} - \Phi^{(0)} \tag{2.3}$$

$$\Delta^2\Phi^{(2)} + \lambda^{(0)}\Delta\Phi^{(2)} + T^{(2)\prime} = H\Phi^{(1)} - \lambda^{(2)}\Delta\Phi^{(0)}$$

$$P^{-1}\Delta T^{(2)} + \lambda^{(0)}T^{(2)} = -v_0T^{(1)} - \Phi^{(1)} - \lambda^{(2)}T^{(0)} \tag{2.4}$$

The boundary conditions follow from (1.3)

$$\Phi^{(n)}(\pm 1) = \Phi^{(n)\prime}(\pm 1) = T^{(n)}(\pm 1) = 0 \tag{2.5}$$

3. The equations of the zero order approximation (2.2) describe the perturbations when $a = 0$, i.e. for uniform temperature planes. The structure of the equations enables one to distinguish two types of perturbations: 1) isothermal, for which $T^{(0)} = 0$, and 2) nonisothermal, corresponding to $T^{(0)} \neq 0$.

For the amplitudes and decrements of the isothermal perturbations we have

$$\Delta^2\varphi^{(0)} + \mu^{(0)} \Delta\varphi^{(0)} = 0, \qquad \varphi^{(0)}(\pm 1) = \varphi^{(0)'}(\pm 1) = 0 \tag{3.1}$$

Equation (3.1) shows the spectra of the eigenfunctions to be defined by the property evenness with a discrete spectrum of eigenvalues $\mu_i^{(0)}$. The normalized even solutions have the form

$$\varphi_i^{(0)} = \frac{\cosh kx}{\cosh k} - \frac{\cosh \xi_i^{(0)}x}{\cosh \xi_i^{(0)}} \qquad (i = 0, 2, 4, \ldots) \qquad (\xi_i^{(0)2} = k^2 - \mu_i^{(0)} < 0) \tag{3.2}$$

The eigenvalues $\mu_i^{(0)}$ are found from the characteristic equation

$$k \tanh k - \xi_i^{(0)} \tanh \xi_i^{(0)} = 0 \quad (i = 0, 2, 4, \ldots) \tag{3.3}$$

If in (3.2) we replace cosh by sinh and in (3.3) tanh by coth, then we obtain the odd solutions $\varphi_i^{(0)}$ with the spectrum $\mu_i^{(0)}$ $(i = 1, 3, 5, \ldots)$.

All decrements $\mu_i^{(0)}$ are real and positive [3].

They do not depend on the Prandtl number and are determined solely by the wave number k. For all k the decrements increase in the sequence $\mu_0^{(0)}$, $\mu_1^{(0)}, \mu_2^{(0)} \ldots$ The eigenfunctions satisfy the condition of orthogonality

$$\int_{-1}^{1} \varphi_i^{(0)} \Delta\varphi_k^{(0)} dx = 0 \qquad (i \neq k) \tag{3.4}$$

Nonisothermal perturbations are described by the system of equations

$$\Delta^2\Phi^{(0)} + \nu^{(0)}\Delta\Phi^{(0)} - T^{(0)'} = 0, \qquad P^{-1}\Delta T^{(0)} + \nu^{(0)}T^{(0)} = 0 \tag{3.5}$$

with boundary conditions (2.5). The spectrum of the decrements $\nu_i^{(0)}$ and the perturbations of temperature $T_i^{(0)}$ is determined from the equation of heat conduction (3.5)

$$\nu_i^{(0)} = P^{-1} [{}^1/_4\pi^2 (i + 1)^2 + k^2] \quad (i = 0, 1, 2, \ldots)$$

$$T_i^{(0)} = \begin{cases} \cos \rho_i^{(0)}x & (i = 0, 2, 4, \ldots) \\ \sin \rho_i^{(0)}x & (i = 1, 3, 5, \ldots) \end{cases} \qquad \rho_i^{(0)2} = P\nu_i^{(0)} - k^2 > 0 \tag{3.6}$$

The perturbations of temperature are orthogonal

$$\int_{-1}^{1} T_i^{(0)} T_k^{(0)} dx = 0 \qquad (i \neq k) \tag{3.7}$$

Because of the presence of convective forces the temperature perturbations lead to the occurrence of velocity perturbations. The amplitudes of the velocity perturbations $\Phi_i^{(0)}$ are found from (3.5) with known $T_i^{(0)}$ and $\nu_i^{(0)}$ given by Formulas (3.6). Even perturbations of temperature correspond to the odd functions $\Phi_i^{(0)}$ $(i = 0, 2, 4, \ldots)$ (3.8)

$$\Phi_i^{(0)} = \frac{(-1)^i \rho_i^{(0)} \sin \rho_i^{(0)}}{P(P-1)\nu_i^{(0)2}} \left[\frac{q_i^{(0)} \cosh q_i^{(0)} \sinh kx - k \cosh k \sinh q_i^{(0)}x}{k \cosh k \sinh q_i^{(0)} - q_i^{(0)} \sinh k \cosh q_i^{(0)}} - \frac{\sin \rho_i^{(0)}x}{\sin \rho_i^{(0)}} \right] \quad (P \neq 1)$$

$$\Phi_i^{(0)} = \frac{1}{2\nu_i^{(0)}} \left[\frac{(-1)^i}{k \cosh k} (\rho_i^{(0)} \sin \rho_i^{(0)} \sinh kx - \rho_i^{(0)} \sinh k \sin \rho_i^{(0)} x) + x \cos \rho_i^{(0)}x \right] \quad (P = 1)$$

$$\left(q_i^{(0)2} = k^2 - \nu_i^{(0)}\right)$$

If in (3.8) we make the substitution sin ⇄ cos and sinh ⇄ cosh, then we obtain the even functions $\Phi_i^{(0)}$ $(i = 1, 3, 5 \ldots)$, corresponding to the odd perturbations of temperature.

Thus, in fluid at rest there occur monotonously decaying perturbations of two types: isothermal, the spectrum of the decrements of which does not depend on the Prandtl number, and nonisothermal, the decrements of which decrease as the Prandtl number grows, proportionally to $1/P$. When $P = 1$ ($k \neq 0$), nonisothermal and isothermal levels alternate in the spectrum in the order $\nu_0^{(0)}, \mu_0^{(0)}, \nu_1^{(0)}, \mu_1^{(0)}, \nu_2^{(0)}, \mu_2^{(0)}, \ldots$ (Fig. 2 c, $G = 0$). With the decrease or increase of the number P there occurs a rarefication or concentration of the spectrum of ν-levels for the fixed position of the μ-levels.

Thus, when $P \ll 1$ the lower part of the spectrum is formed by the isothermal levels and when $P \gg 1$, the lower levels are nonisothermal (Fig. 2 b, d ; $G = 0$).

4. Let us study the behavior of perturbations in the moving fluid for a small difference of temperature between the planes. The corrections to the eigenfunctions of the zero order approximation will be sought in the form of expansions with respect to the complete system of functions $\{T_i^{(0)}\}$ and $\{\varphi_i^{(0)}\}$

$$T_i^{(n)} = \sum_k a_{ik}^{(n)} T_k^{(0)}, \qquad \Phi_i^{(n)} = \sum_k b_{ik}^{(n)} \varphi_k^{(0)} \tag{4.1}$$

We shall indicate the coefficients of the expansions (4.1), necessary for the calculation of the second order corrections of the "isothermal" and "nonisothermal" perturbations: $\mu_i^{(2)}$ and $\nu_i^{(2)}$

To find $\mu_i^{(2)}$ it is necessary to know $T_i^{(1)}$, $\Phi_i^{(1)}$ and $T_i^{(2)}$. The corresponding coefficients of the expansion are found by the usual method from the equations of the successive approximations (2.3), (2.4)

$$a_{ik}^{(1)} = -\frac{C_{ki}}{\mu_i^{(0)} - \nu_k^{(0)}}, \qquad b_{ik}^{(1)} = \frac{1}{\mu_i^{(0)} - \mu_k^{(0)}}\left(H_{ki} + \sum_l \frac{C_{li} D_{lk}}{\mu_i^{(0)} - \nu_l^{(0)}}\right) \tag{4.2}$$

$$a_{ik}^{(2)} = \frac{1}{\mu_i^{(0)} - \nu_k^{(0)}}\left[\sum_l \frac{C_{li} B_{lk}}{\mu_i^{(0)} - \nu_l^{(0)}} - \sum_{m\neq i} \frac{C_{km}}{\mu_i^{(0)} - \mu_m^{(0)}}\left(H_{mi} + \sum_l \frac{C_{li} D_{lm}}{\mu_i^{(0)} - \nu_l^{(0)}}\right)\right]$$

Here we have introduced the following notation for the matrix elements:

$$H_{mn} = \frac{1}{J_m}\int \varphi_m^{(0)} H \varphi_n^{(0)} dx, \qquad D_{mn} = \frac{1}{J_n}\int T_m^{(0)\prime} \varphi_n^{(0)} dx$$

$$C_{mn} = \frac{1}{Y_m}\int T_m^{(0)} \varphi_n^{(0)} dx, \qquad B_{mn} = \frac{1}{Y_m}\int T_m^{(0)} v_0 T_n^{(0)} dx \tag{4.3}$$

$$J_m = \int \varphi_m^{(0)} \Delta \varphi_m^{*(0)} dx, \qquad Y_m = \int T_m^{(0)2} dx$$

Integration everywhere is within the range from -1 to 1. The coefficients $b_{ii}^{(1)}$ are equal to zero, which follows from the condition of normalcy in the case of the odd unperturbed velocity profile.

For the second order corrections $\mu_i^{(2)}$ we obtain Formula

$$\mu_i^{(2)} = \sum_{m\neq i} \frac{H_{im} H_{mi}}{\mu_i^{(0)} - \mu_m^{(0)}} + \sum_{m\neq i}\sum_n \frac{H_{im} C_{ni} D_{nm} + H_{mi} D_{ni} C_{nm}}{(\mu_i^{(0)} - \mu_m^{(0)})(\mu_i^{(0)} - \nu_n^{(0)})} -$$

$$-\sum_n \sum_l \frac{D_{ni} C_{li}}{(\mu_i^{(0)} - \nu_n^{(0)})(\mu_i^{(0)} - \nu_l^{(0)})}\left(B_{nl} - \sum_{m\neq i} \frac{C_{nm} D_{lm}}{\mu_i^{(0)} - \mu_m^{(0)}}\right) \tag{4.4}$$

Similar formulas are obtained also for "nonisothermal" perturbations

$$b_{ik}^{(0)} = -\frac{D_{ik}}{\nu_i^{(0)} - \mu_k^{(0)}}, \qquad a_{ik}^{(1)} = \frac{1}{\nu_i^{(0)} - \nu_k^{(0)}}\left(-B_{ki} + \sum_l \frac{D_{il} C_{kl}}{\nu_i^{(0)} - \mu_l^{(0)}}\right) \tag{4.5}$$

$$b_{ik}^{(1)} = -\frac{1}{\nu_i^{(0)} - \mu_k^{(0)}}\left[\sum_l \frac{D_{il} H_{kl}}{\nu_i^{(0)} - \mu_l^{(0)}} + \sum_{m\neq i} \frac{D_{mk}}{\nu_i^{(0)} - \nu_m^{(0)}}\left(-B_{mi} + \sum_l \frac{D_{il} C_{ml}}{\nu_i^{(0)} - \mu_l^{(0)}}\right)\right]$$

$$\nu_i^{(2)} = \sum_{m\neq i} \frac{B_{im}B_{mi}}{\nu_i^{(0)} - \nu_m^{(0)}} - \sum_{m\neq i}\sum_{n} \frac{B_{im}D_{in}C_{mn} + B_{mi}C_{in}D_{mn}}{(\nu_i^{(0)} - \nu_m^{(0)})(\nu_i^{(0)} - \mu_n^{(0)})} +$$
$$+ \sum_{n}\sum_{l} \frac{C_{in}D_{il}}{(\nu_i^{(0)} - \mu_n^{(0)})(\nu_i^{(0)} - \mu_l^{(0)})} \left(H_{nl} + \sum_{m\neq i} \frac{D_{mn}C_{ml}}{\nu_i^{(0)} - \nu_m^{(0)}}\right) \qquad (4.6)$$

where $a_{i\,i}^{(1)} = 0$ from the condition of normalcy, and all the matrix elements are determined by Formulas (4.3). Explicit expressions for the matrix elements are cumbersome and are not presented here.

Summation is carried out with respect to the unperturbed μ- and ν-levels. These sums in the corrections to the given level characterize its interaction with the isothermal and nonisothermal levels respectively.

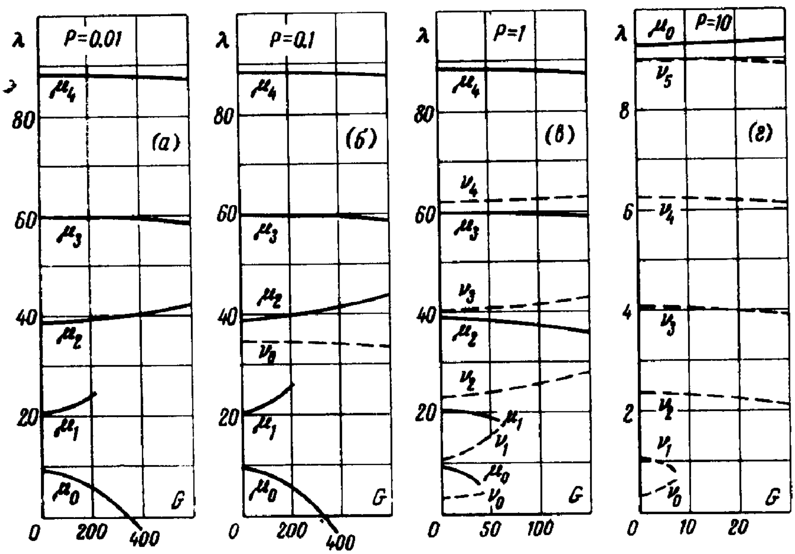

Fig. 2

In the limiting case with $P \to 0$ the decrements $\nu_i^{(0)} \to \infty$ and the corrections to the "isothermal" perturbations and their decrements pass over to the corresponding formulas of [3] (for the case of the odd profile)

Formulas (2.1) and (4.1) to (4.6) enable one to find the spectrum of the decrements and the eigenfunctions of the perturbations for small Grasshof numbers.

For definite values of the parametrs P and k it can happen that "degeneracy" occurs in the unperturbed spectrum, i.e. coincidence of the decrements of the isothermal and nonisothermal perturbations, $\nu_i^{(0)} = \mu_k^{(0)}$. The corrections to such a degenerate level cannot be found by means of the expansions considered; as is clear from the formulas presented, they lose significance in the case of degeneracy.

5. The second order corrections to the decrements were computed on the "ARAGATS" computer in the approximation of 24 (12 + 12) basic functions. Fig. 2 shows the spectra of the decrements taking into account the second order corrections with $k = 1$. The continuous lines indicate the "isothermal" levels, and the dashed lines the "nonisothermal" (*).

*) Here one should stress the conventionality of the term "isothermal" level, since with $G \neq 0$ this level corresponds to a nonzero perturbation of temperature (See (4.2)).

For small values of Prandtl number P (Fig.2 a, b) the lower part of the spectrum is occupied by levels of "isothermal" perturbations. "Nonisothermal" perturbations exert on them an insignificant influence. Formally this is expressed by the fact that the sums with respect to the ν-levels in Formula (4.4) give small contributions to the correction to the μ-levels. In essence, the $\mu_i^{(2)}$ is determined for small P only by the first sum in the right-hand side; this sum describes the interaction of the ith level with the other isothermal levels. The spectrum in Fig.2a almost coincides with the spectrum of the hydrodynamic problem with a given steady velocity profile [3].

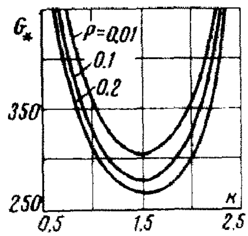

Fig. 3

With increase of Prandtl number P the form of the spectrum changes significantly (Fig.2 c, d). When $P=1$ the levels of different type alternate in the spectrum, so the mutual interaction of the "isothermal" and the "nonisothermal" perturbations here begins to be determinative. In Fig.2c it is clear that with increase of G the stability of the "isothermal" perturbations decreases, whilst that of the "nonisothermal" increases.

For large values of number P (Fig.2d) "nonisothermal" levels are located in the lower part of the spectrum. In contrast to the case of small values of P, a significant influence on the behavior of the lower levels is exerted by the levels of the other type, i.e. the "isothermal" ones. This is easy to establish from the formula for the second order corrections $\nu_i^{(2)}$ (4.6), where the sums with respect to the μ-levels remain substantial for large P as well.

In the case of small values of P (Fig.2 a, b) the lower "isothermal" level can be extended to the intersection with the G-axis. The point of intersection enables us to find approximately the Grasshof number for the neutral perturbation with a given value of k. In Fig.3 we show the neutral curves obtained in this way for different values of the number P. The critical Grasshof numbers are close to those obtained in [2].

6. The expansions we have considered do not enable us to study the intersection in the spectrum of perturbations; moreover, as indicated, the expansions lose their meaning, when degeneracy occurs in the unperturbed spectrum. To elucidate the character of the spectrum in these cases we can employ Galerkin's method, and as basic functions $\varphi^{(0)}$ and $T^{(0)}$ are convenient.

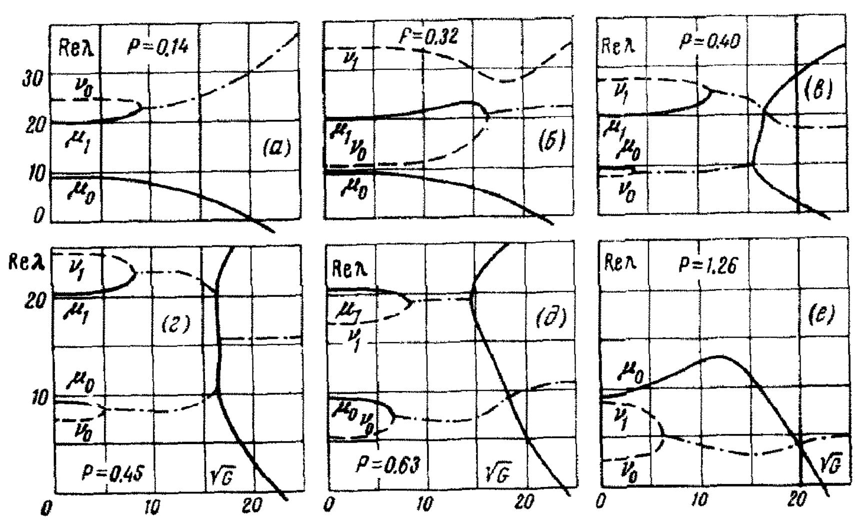

Fig. 4

The approximation

$$\Phi = c_0\varphi_0^{(0)} + c_1\varphi_1^{(0)}, \qquad T = d_0T_0^{(0)} + d_1T_1^{(0)} \tag{6.1}$$

gives the exact values of the four decrements $\mu_0^{(0)}$, $\mu_1^{(0)}$, $\nu_0^{(0)}$ and $\nu_1^{(0)}$ when $G = 0$. When $G \neq 0$ we obtain from the approximate solution at least a qualitative picture of the intersection of four levels considered.

Fig.4 shows the spectra of the decrements obtained in the approximation (6.1) for wave number $k = 1$ and certain values of the Prandtl number in the interval $0.14 \leq P \leq 1.26$ (in this interval the intersecting levels are lower in the spectrum). With increase of number P the distribution of levels changes, and for certain values of P degeneracy occurs for the unperturbed $\nu^{(0)}$- and $\mu^{(0)}$- levels. Fig.4 shows the real μ- and ν-levels, starting from the axis $G = 0$, indicated by full and dashed curves, respectively. At the confluence of the real levels a pair of complex conjugate decrements is formed, describing oscillatory perturbations. The common real part of those decrements is depicted by chain-dotted curves.

As is clear from Fig.4, the spectrum does not show "simple" intersections of real levels. The confluence of real levels leads to the formation of certain critical values of G of a pair of complex-conjugate decrements. Moreover, at the intersection of μ- and ν-levels it is possible also to have a case of "splitting" of a complex conjugate pair into two real levels as the parameter G achieves a second critical value (Fig.4 c,d,e). Special points of this sort do not occur in spectra of perturbations of isothermal flows; apparently they are specific to convective problems. In Fig.4d one can see how two secondary real levels again combine, forming a pair of oscillatory perturbations.

Degeneracy of the unperturbed spectrum corresponds to values of the parameters for which special points occur on the axis $G = 0$. So, in the three cases of degeneracy $(\mu_1^{(0)} = \nu_0^{(0)}, \mu_0^{(0)} = \nu_0^{(0)}, \mu_1^{(0)} = \nu_1^{(0)})$ the pair of complex-conjugate decrements arises for an arbitrarily small value of G. In the case $\mu_0^{(0)} = \nu_1^{(0)}$ the degenerate level for arbitrarily small G splits into two real ones.

From Fig.4 it is clear that in the interval of values of the Prandtl number P under consideration there is monotonous instability; moreover, its onset is connected with "isothermal" perturbations; the axis of G is intersected either by a real μ-level, or by one of the real levels formed by "decomposition" of a complex conjugate pair. It is interesting that although for a change of the number P in the interval of P under consideration the spectrum changes form rather radically, the critical value of the Grasshof number, which determines the neutral perturbation, varies only slightly.

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BIBLIOGRAPHY

1. Gershuni, G.E., Ob ustoichivosti ploskogo konvectivnogo dvizheniia zhidkosti (On the stability of plane convective motion of a liquid). Zh.tekh.Fiz, Vol.23, № 10, 1953.

2. Gershuni, G.E. and Zhukhovitskii, E.M., O dvukh tipakh neustoichivosti konvektivnogo dvizheniia mezhdu parallel'nymi vertikal'nymi ploskostiami (On two types of instability of convective motion between parallel vertical planes). Izv.vyssh.ucheb.Zaved., Fizika, № 4,1958.

3. Birikh, R.V., Gershuni, G.E. and Zhukhovitskii, E.M., O spektre vozmushchenii ploskoparallel'nykh techenii pri malykh chislakh Reinol'dsa (On the spectrum of perturbations of plane-parallel flows at low Reynolds numbers). *PMM* Vol.29, № 1, 1965.

4. Landau, L.D. and Lifshits, E.M., Mekhanika sploshnykh sred (Mechanics of Continuous Media). Gostekhizdat, 1954.

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